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Similarity Measures of Intuitionistic Fuzzy Sets

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Abstract

In this paper, we propose two similarity measures for measuring the degree of similarity between intuitionistic fuzzy sets.

Keywords: Fuzzy Sets, Intuitionistic Fuzzy Sets, Similarity Measure.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper ([6]) in 1965. In 1975, in another direction, Atanassov ([1]) introduced the concept of intuitionistic fuzzy set. Many measures of similarity between fuzzy sets have been proposed in ([7], [3]). In ([5]), S.M. Chen, proposed two methods for measures of similarity between vague sets. Also, using vector approach, Chen ([2], [4]) defined a measure of similarity between two fuzzy sets.

In the present paper, we propose some methods for measures of similarity between intuitionistic fuzzy sets of same nature of S.M. Chen's ([5]).

Intuitionistic Fuzzy Sets

Let a set X be fixed. An IFS A_i in X consists the performance value P_{ij}, for fixed i and j = 1, 2, 3, … having the form

$$A_i = \left\{\left\langle x_j, \mu_{A_i}(x_j), \nu_{A_i}(x_j)\right\rangle : \forall x_j \in X\right\}$$

$$= \left\{\left\langle \mu_{ij}, \nu_{ij}\right\rangle : j=1,2,3,\ldots\right\}.$$

Where the performance value $P_{ij} = \langle \mu_{ij}, \nu_{ij}\rangle, j=1,2,,\ldots$ and the function $\mu_{ij} : X \to [0,1]$ and $\nu_{ij} : X \to [0,1]$ defines the degree of membership and degree of non-membership respectively of the element $x_j \in X$ to the set A_i, which is a subset of X and for every $x_j \in X, 0 \le \mu_{ij} + \nu_{ij} \le 1$.

It is clear that, A performance value $P_{ij} = \langle \mu_{ij}, \nu_{ij}\rangle$ consists of the membership value μ_{ij} and the non-membership value ν_{ij}. Clearly the hesitation part is

$$\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$$

we use the following notation.

$$\mu(P_{ij}) = \mu_{ij},\ \nu(P_{ij}) = \nu_{ij} \text{ and } \pi(P_{ij}) = \pi_{ij}$$

Example 1.1: Consider two IFSs A_1 and A_2 of $X = \{x_1, x_2, x_3, x_4\}$ given by the following table

X	$\langle \mu_{A_1}, \nu_{A_1} \rangle$	$\langle \mu_{A_2}, \nu_{A_2} \rangle$
x_1	.60, .30	.65, .25
x_2	.70, .20	.75, .20
x_3	.80, .12	.31, .44
x_4	.90, .02	.52, .32

That is

$$A_1 = \{(.60,.30)/x_1, (.70,.20)/x_2, (.80,.12)/x_3, (.90,.02)/x_4\}$$

$$A_2 = \{(.65,.25)/x_1, (.75,.20)/x_2, (.31,.44)/x_3, (.52,.32)/x_4\}$$

that is the performance values are

$$P_{11} = (.60,.30) = (\mu_{11}, \nu_{11}) = (\mu_{A_1}(x_1), \nu_{A_1}(x_1))$$

$$P_{12} = (.70,.20) = (\mu_{12}, \nu_{12}) = (\mu_{A_1}(x_2), \nu_{A_1}(x_2))$$

$$P_{13} = (.80,.12) = (\mu_{13}, \nu_{13}) = (\mu_{A_1}(x_3), \nu_{A_1}(x_3))$$

$$P_{14} = (.90,.02) = (\mu_{14}, \nu_{14}) = (\mu_{A_1}(x_4), \nu_{A_1}(x_4))$$

$$P_{21} = (.65,.25) = (\mu_{21}, \nu_{21}) = (\mu_{A_2}(x_1), \nu_{A_2}(x_1))$$

$$P_{22} = (.75,.20) = (\mu_{22}, \nu_{22}) = (\mu_{A_2}(x_2), \nu_{A_2}(x_2))$$

$$P_{23} = (.31,.44) = (\mu_{23}, \nu_{23}) = (\mu_{A_2}(x_3), \nu_{A_2}(x_3))$$

$$P_{24} = (.52,.32) = (\mu_{24}, \nu_{24}) = (\mu_{A_2}(x_4), \nu_{A_2}(x_4))$$

2. Similarity Measure:

Let $x = (\mu_x, \nu_x)$ be a performance value, where $\mu_x \in [0,1]$ and $\nu_x \in [0,1]$, $0 \le \mu_x + \nu_x \le 1$. Then the measure of 'x' can be evaluated by a measure function 'm' shown as follows:

(2.1) $$m(x) = 2\mu_x + \pi_x - 1$$

where $\pi_x = 1 - \mu_x - \nu_x$ be the hesitation part and $m(x) \in [-1,1]$

Let x and y be two performance values, $x = (\mu_x, \nu_x)$ and $y = (\mu_y, \nu_y)$, then the degree of similarity between the performance values x and y can be evaluated by the function 'S' as

(2.2) $$S(x.y) = 1 - \left| \frac{m(x) - m(y)}{2} \right|$$

where $m(x) = 2\mu_x + \pi_x - 1$ and $m(y) = 2\mu_y + \pi_y - 1$

Case-I If the performance values x = (1, 0) and y = (0, 1), then we can see that

$$m(x) = 2.1 + 0 - 1 = 1 \text{ and } m(y) = 2 \cdot 0 + 0 - 1 = -1$$

By applying equation 2.2, the degree of similarity between x and y can be evaluated and is equal to

$$S(x.y) = 1 - \left| \frac{1-(-1)}{2} \right| = 1 - 1 = 0.$$

Case-II If the performance values x = (1, 0) and y = (1, 0), then we can see that

$$m(x) = 1 \text{ and } m(y) = 1.$$

So, degree of similarity between x and y can be evaluated and is equal to

$$S(x.y) = 1 - \left| \frac{1-1}{2} \right| = 1 - 0 = 1.$$

Case-III If the performance values x = (0, 1) and y = (0, 1), then we can see that

$$m(x) = -1 \text{ and } m(y) = -1.$$

So, the degree of similarity between x and y can be evaluated and is equal to

$$S(x.y) = 1 - \left| \frac{-1+1}{2} \right| = 1 - 0 = 1.$$

Case-IV If the performance values x = (0, 1) and y = (1, 0), then we can see that

$$m(x) = -1 \text{ and } m(y) = 1.$$

So, the degree of similarity between x and y can be evaluated and is equal to

$$S(x.y) = 1 - \left| \frac{-1-1}{2} \right| = 1 - 1 = 0.$$

It is obvious that if x and y are identical performance value, then m(x) = m(y) and the degree of similarity between the performance values x and y is equal to 1.

Let A_1 and A_2 be two IFSs in $X = \{x_1, x_2, x_3 .... x_n\}$. Then

$$A_1 = \{(\mu_{A_1}(x_1), \nu_{A_1}(x_1))/x_1, (\mu_{A_1}(x_2), \nu_{A_1}(x_2))/x_2, ... (\mu_{A_1}(x_n), \nu_{A_1}(x_n))/x_n\}$$

$$A_2 = \{(\mu_{A_2}(x_1), \nu_{A_2}(x_1))/x_1, (\mu_{A_2}(x_2), \nu_{A_2}(x_2))/x_2, ... (\mu_{A_2}(x_n), \nu_{A_2}(x_n))/x_n,\}$$

So that

$$P_{1i} = \left(\mu_{A_1}(x_i), \nu_{A_1}(x_i)\right) = \left(\mu_{A_{1i}}, \nu_{A_{1i}}\right),\ i = 1, 2, \dots n$$

and $$P_{2i} = \left(\mu_{A_2}(x_i), \nu_{A_2}(x_i)\right) = \left(\mu_{A_{2i}}, \nu_{A_{2i}}\right),\ i = 1, 2, \dots n$$

be the performance values of 'x_i' in the IFSs A_1 and A_2 respectively, then applying equation 2.1, we can see that

$$\text{(2.3)} \qquad \begin{aligned} m(P_{1i}) &= 2\mu_{A_1}(x_i) + \pi_{A_1}(x_i) - 1 \text{ and} \\ m(P_{2i}) &= 2\mu_{A_2}(x_i) + \pi_{A_2}(x_i) - 1,\ 1 \le i \le n \end{aligned}$$

and the degree of similarity between the IFSs A_1 and A_2 can be evaluated by the function T, as

$$\text{(2.4)} \qquad T(A_1, A_2) = \frac{1}{n}\sum_{i-1}^{n} S(A_1, A_2) = \frac{1}{n}\sum_{i-1}^{n}\left(1 - \left|\frac{m(P_{1i}) - m(P_{2i})}{2}\right|\right)$$

where $T(A_1, A_2) \in [0,1]$. The larger value of $T(A_1, A_2)$, the more the similarity between the IFSs A_1 and A_2.

Example 2.1: Let $X = \{x_1, x_2, x_3, x_4\}$ be a fixed set and

$$A_1 = \{(.60,.30)/x_1, (.70,.20)/x_2, (.80,.12)/x_3, (.90,.02)/x_4\}$$

$$A_2 = \{(.65,.25)/x_1, (.75,.20)/x_2, (.31,.44)/x_3, (.52,.32)/x_4\}$$

where

$$P_{11} = (.60,.30) = (\mu_{11}, \nu_{11}) = \left(\mu_{A_1}(x_1), \nu_{A_1}(x_1)\right)$$

$$P_{12} = (.70,.20) = (\mu_{12}, \nu_{12}) = \left(\mu_{A_1}(x_2), \nu_{A_1}(x_2)\right)$$

$$P_{13} = (.80,.12) = (\mu_{13}, \nu_{13}) = \left(\mu_{A_1}(x_3), \nu_{A_1}(x_3)\right)$$

$$P_{14} = (.90,.02) = (\mu_{14}, \nu_{14}) = \left(\mu_{A_1}(x_4), \nu_{A_1}(x_4)\right)$$

$$P_{21} = (.65,.25) = (\mu_{21}, \nu_{21}) = \left(\mu_{A_2}(x_1), \nu_{A_2}(x_1)\right)$$

$$P_{22} = (.75,.20) = (\mu_{22}, \nu_{22}) = \left(\mu_{A_2}(x_2), \nu_{A_2}(x_2)\right)$$

$$P_{23} = (.31,.44) = (\mu_{23}, \nu_{23}) = \left(\mu_{A_2}(x_3), \nu_{A_2}(x_3)\right)$$

$$P_{24} = (.52,.32) = (\mu_{24}, \nu_{24}) = \left(\mu_{A_2}(x_4), \nu_{A_2}(x_4)\right)$$

Now applying equation 2.1, we can get

$$m(P_{11}) = 2\mu_{A_1}(x_1) + \pi_{A_1}(x_1) - 1 = 2 \times .60 + .10 - 1 = .30$$

$$m(P_{12}) = 2\mu_{A_1}(x_2)+\pi_{A_1}(x_2) - 1 = 2\times.70+.10-1 = .50$$

$$m(P_{13}) = 2\mu_{A_1}(x_3)+\pi_{A_1}(x_3) - 1 = 2\times.80+.08-1 = .68$$

$$m(P_{14}) = 2\mu_{A_1}(x_4)+\pi_{A_1}(x_4) - 1 = 2\times.90+.08-1 = .88$$

$$m(P_{21}) = 2\mu_{A_2}(x_1)+\pi_{A_2}(x_1) - 1 = 2\times.65+.10-1 = .40$$

$$m(P_{22}) = 2\mu_{A_2}(x_2)+\pi_{A_2}(x_2) - 1 = 2\times.75+.05-1 = .55$$

$$m(P_{23}) = 2\mu_{A_2}(x_3)+\pi_{A_2}(x_3) - 1 = 2\times.31+.25-1 = -.13$$

$$m(P_{24}) = 2\mu_{A_2}(x_4)+\pi_{A_2}(x_4) - 1 = 2\times.52+.16-1 = .20$$

By applying equation 2.4, the degree of similarity between A_1 and A_2 can be evaluated by

$$T(A_1, A_2) = \frac{1}{n}\sum_{i-1}^{n} S(A_1, A_2) = \frac{1}{4}\sum_{i-1}^{4}\left(1-\left|\frac{m(P_{1i})-m(P_{2i})}{2}\right|\right)$$

$$= \frac{1}{4}[.95+.975+.595+.66] = .795$$

which indicate that the degree of similarity between the IFSs A_1 and A_2 be .795.

In the following, we present a weighted similarity measure between IFSs A_1 and A_2. Let A_1 and A_2 be two IFSs $X = \{x_1, x_2, x_3 ... x_n\}$. Then

$$A_1 = \sum_{i=1}^{n} (\mu_{A_1}(x_i), \nu_{A_1}(x_i))/x_i = \sum_{i=1}^{n} P_{1i}/x_i \text{ and}$$

$$A_2 = \sum_{i=1}^{n} (\mu_{A_2}(x_i), \nu_{A_2}(x_i))/x_i = \sum_{i=1}^{n} P_{2i}/x_i$$

Assume that, the weight of the elements x_i in X is w_i respectively, where $w_i \in [0,1]$ and $1 \le i \le n$. Then the degree of similarity between IFSs A_1 and A_2 can be evaluated by the weighting function 'W'

$$W(A_1, A_2) = \sum_{i=1}^{n} w_i^*\left(1-\left|\frac{m(P_{1i})-m(P_{2i})}{2}\right|\right) \Big/ \sum_{i=1}^{n} w_i \tag{2.5}$$

where $W(A_1, A_2) \in [0,1]$. Then the larger value of $W(A_1, A_2)$, the more similarity between the IFSs A_1 and A_2.

Example 2.2: Consider the previous example we have

$m(P_{11}) = .30$ $\qquad$ $m(P_{21}) = .40$

$m(P_{12}) = .50 \qquad m(P_{22}) = .55$

$m(P_{13}) = .68 \qquad m(P_{23}) = -.13$

$m(P_{14}) = .88 \qquad m(P_{24}) = .20$

Assume that the weights of x_1, x_2, x_3 and x_4 be .5, .8, 1.0 and 0.7 respectively. Then applying equation 2.5, the degree of similarity between IFSs A_1 and A_2 can be evaluated by the weighting function W as

$$W(A_1, A_2) = \sum_{i=1}^{n} w_i^* \left(1 - \left|\frac{m(P_{1i}) - m(P_{2i})}{2}\right|\right) \Big/ \sum_{i=1}^{n} w_i$$

$$= \frac{.5\times.95 + .8\times.975 + 1.0\times.595 + .7\times.66}{.5 + .8 + 1.0 + .7}$$

$$= \frac{.475 + .78 + .595 + .462}{3}$$

$$= .770\overline{6}$$

$$= .771$$

3. Conclusion

Although many similarity measure have been proposed in the literature for measuring the degree of similarity between fuzzy sets. In this section we propose similarity measures for measuring the degree of similarity between IFSs. The proposed measures can provide a useful way to deal with the similarity measures of IFSs.

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